Dynamics of dense granular media: flow visualization and continuum modeling

Ashish V. Orpe

av.orpe@ncl.res.in

Chemical Engineering Division

National Chemical Laboratory, Pune

Unilever R&D, Bangalore

May 27, 2011

Acknowledgements

Collaborators

Prof. D. V. Khakhar (IIT Bombay, Powai, Mumbai, India)

Prof. Arshad Kudrolli (Clark University, MA, USA)

Dr. Chris Rycroft (Post-doc, Lawrence Berkeley National Laboratory, Berkeley, CA, USA)

Salome Siavoshi (Former Masters Student at Clark University, Worcester, MA, USA)

Qing Xu (Former Masters Student at Clark University, Worcester, MA, USA)

Mayuresh Kulkarni (Project Associate, National Chemical Laboratory, Pune, India)

Funding Agencies

Department of Science and Technology, India

National Science Foundation, USA

What is granular media ?

Typical properties

❑ Large collection of particles (size range: μm - mm)

❑ Athermal systems ($k_BT \sim 0$) with particle inertia important

❑ Particle-particle and particle-liquid interactions

❑ Interactions are inelastic (highly dissipative systems)

❑ Unique material (exhibits solid/liquid/gas-like behaviour)

❑ Inherent micro-structure (packing fraction $\phi \sim 0.6$ rcp)

Present almost everywhere around us !

(e.g. Avalanches, Land-slides, river-bed-transport, sand-dunes, rings of saturn, salt-shaker, hourglass…..)

Industrial Applications

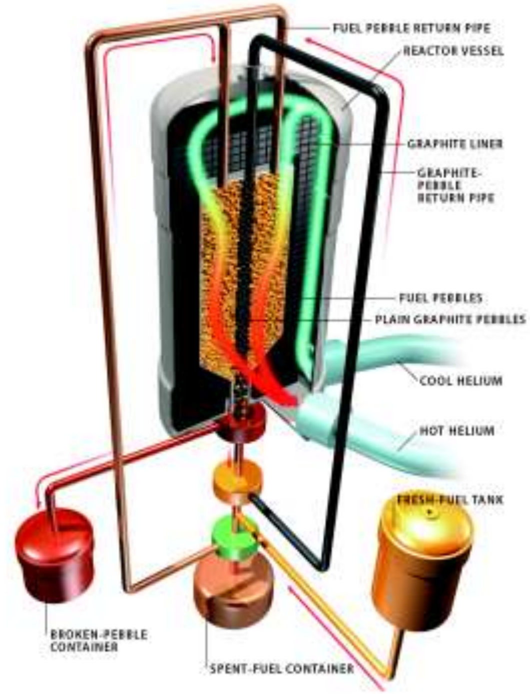
Industrial systems
- Agriculture and food
- Chemicals & petrochemicals
- Construction
- Ceramics
- Mining & minerals
- Pharmaceuticals

Granular matter is second highest by weight of materials processed by man; water is highest !

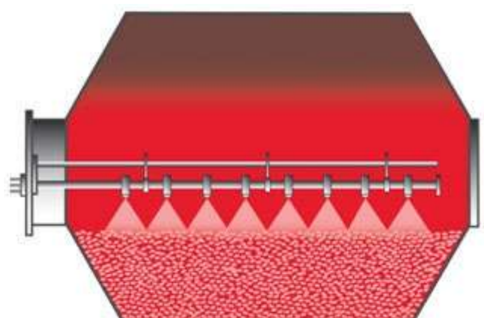
Detailed understanding for better and improved processing of granular material

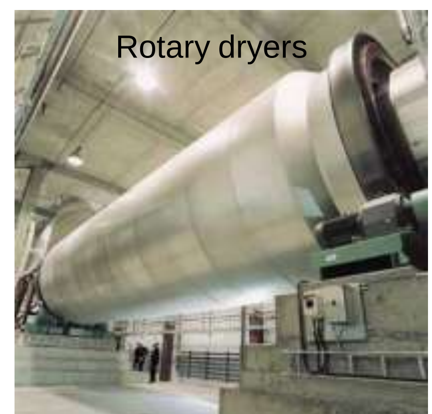

Rotary kiln

Nuclear pebble bed reactors




V-blenders


Tablet coating

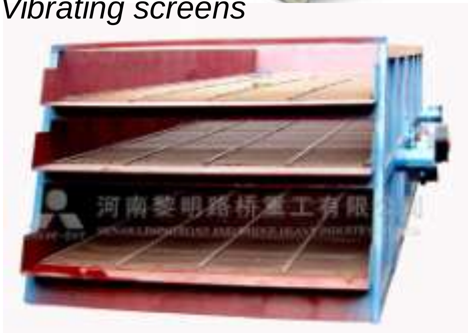

Rotary dryers

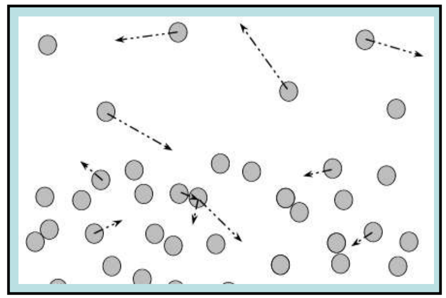
Classification of dry granular flows

Dilute granular systems

Vibrating feeders

Vibrating screens





Particle motion

Inelastic, binary collisions (gas-like)

Kinetic theory formulations

- Jenkins & Savage, Haff (1984)
- Kumaran (1998)

Experiments: Vibro-fluidized and Gas-fluidized granular beds

Molecular dynamics, MC simulations

Dense granular systems

▪ Close packed ($\phi \sim 0.6$): solid-like (crowded particles) and liquid-like (disordered structure). Needs a yield stress to flow

▪ Multi-particle contacts, co-operative flow of particles, flow induced particle rearrangements

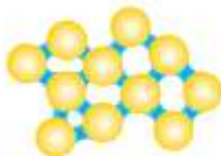
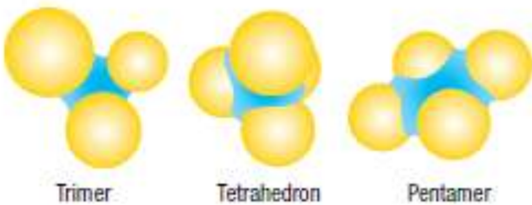
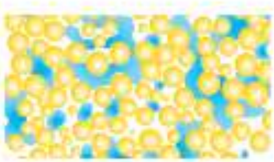
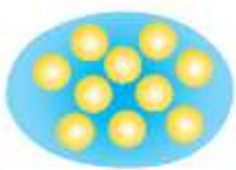
Nuclear reactor drainage

Heap formations

Caging in a Traffic Jam

Interstitial fluid in granular media

Increasing concentration of liquid ↓

	Dry grains — cohesion negligible
	Partially saturated — at small volume fractions, liquid bridges are formed between grains near points of contact. Liquid bridges induce cohesion between grains.
	At higher volume fractions, liquid bridges merge to give trimers, tetrahedra and pentamers.
	At still higher volume fractions, large contiguous wet clusters form.
	Slurry — the pore space is fully saturated with liquid. Cohesion becomes negligible again.

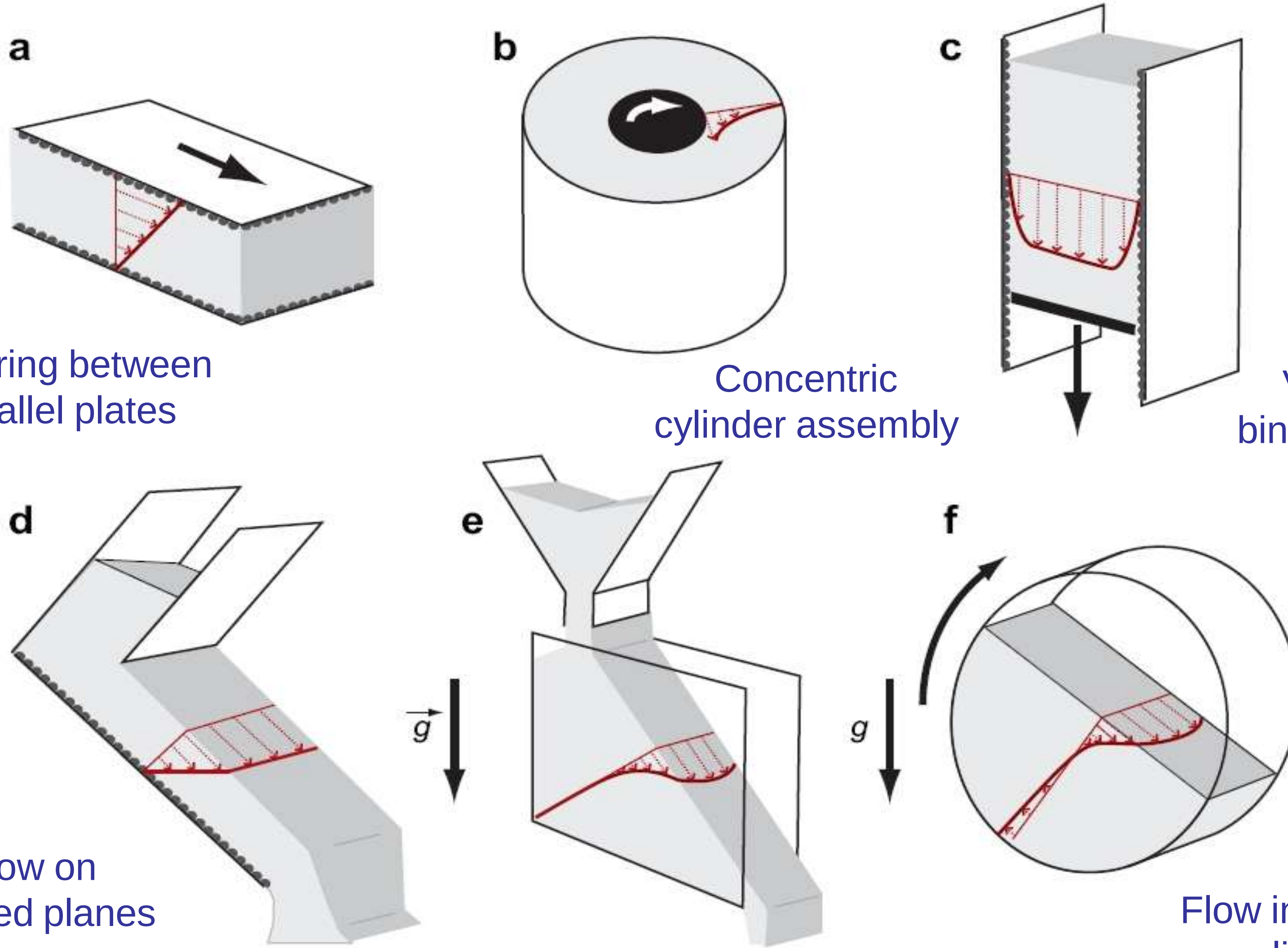
Friction and collision between particles leading to overall flow

- Added resistance to flow due to liquid bridges holding particles
- Additional forces (capillary and lubrication) contributing to overall mechanics
- Modified particle surface properties due to interstitial liquid

Additional drag forces and liquid hydrodynamics

(Kudrolli, Nat. Phys. 2008)

Different geometries to study granular flow



Shearing between parallel plates

Concentric cylinder assembly

Vertical bin/silo flows

Flow on inclined planes

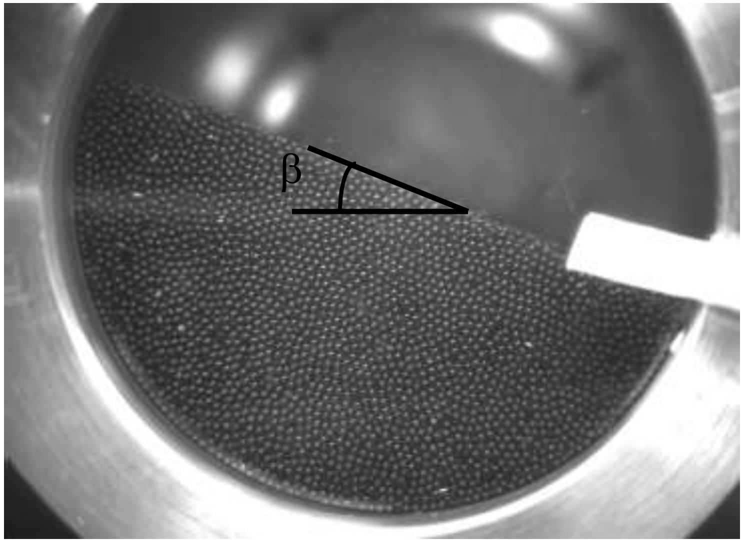
Flow over a heap of solid material

Flow in rotating cylinders

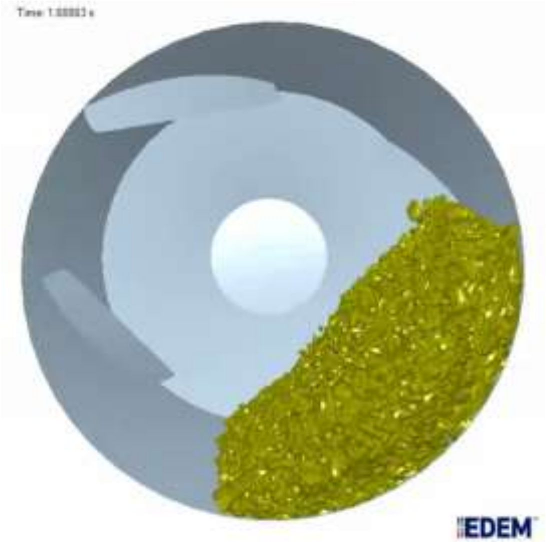
(Forterre & Pouliquen, 2008)

Rotating cylinder flow

Surface granular flow



Flow of spherical glass beads



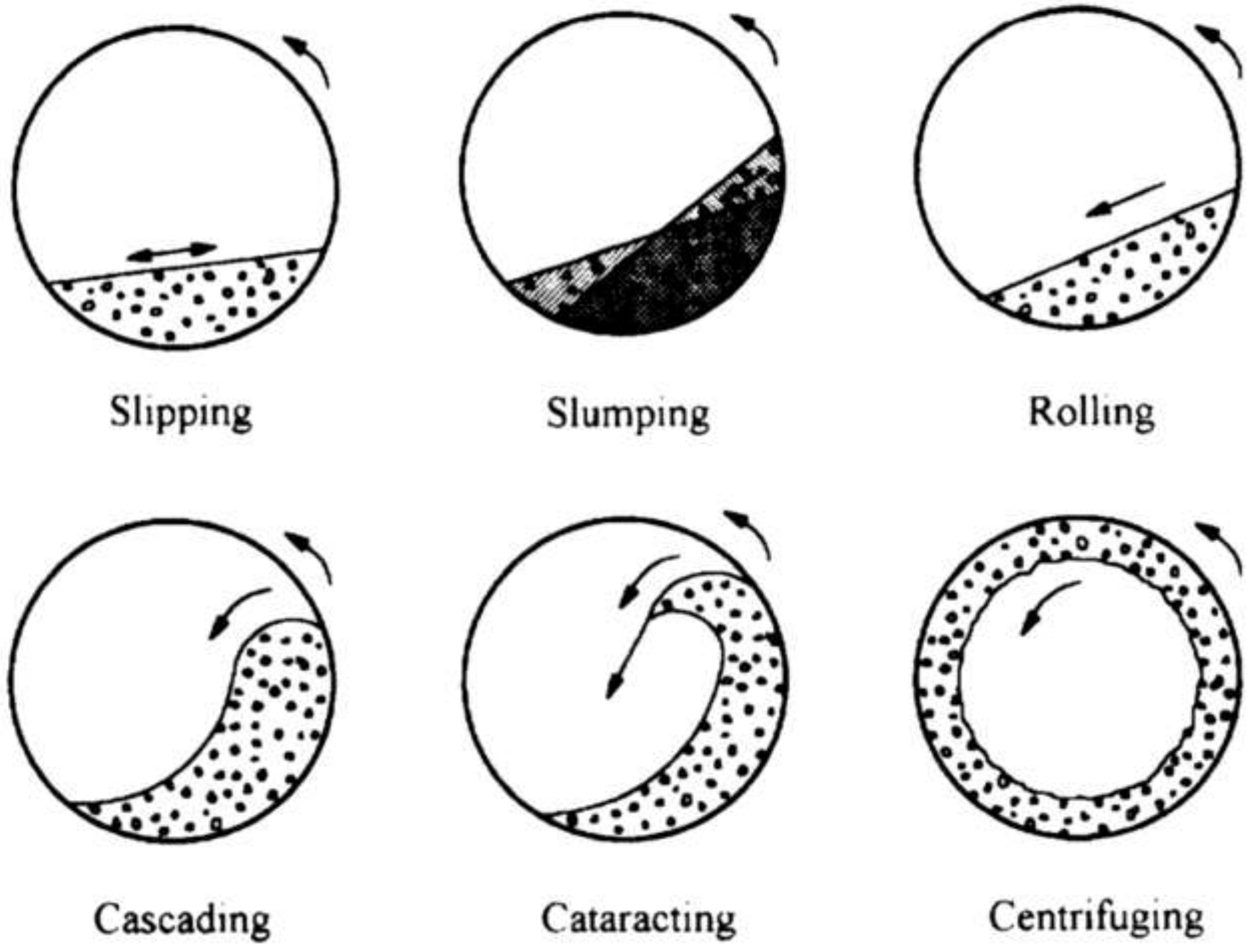
Flow during tablet coating

Typical Characteristics

- Cylinder partially filled with particles with remainder a fluid (air or viscous liquid)
- Flow induced by gravity, controlling parameter is the flow rate
- Flow occurs in a thin layer over a relatively static bed rotating with cylinder
- Shear in the flowing layer due to rotating static bed
- Exchange of particles across shearing boundary
- **Applications: Drum mixers, Tablet pan coaters, Fertilizer drying**

Flow regimes in rotating cylinder

Effect of increasing the rotational speed



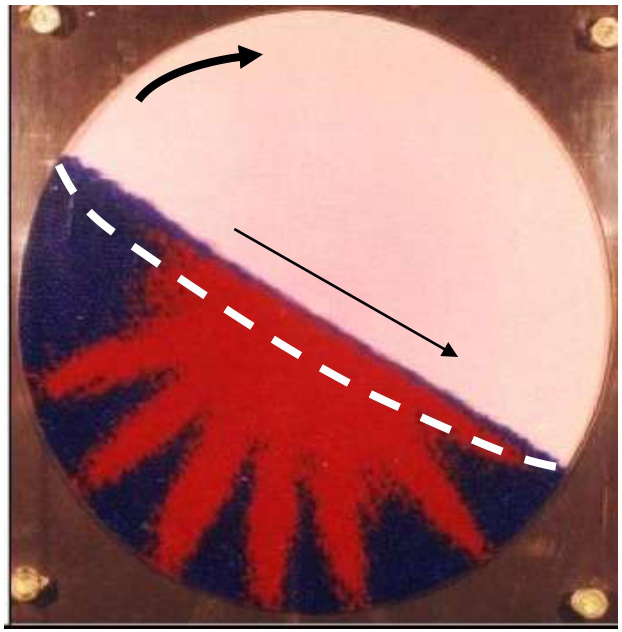
Most industrial operations carried out in steady state rolling regime

(Henein et al. 1984)

Radial segregation in rotation cylinders

Initial state: Well-mixed particles of different size

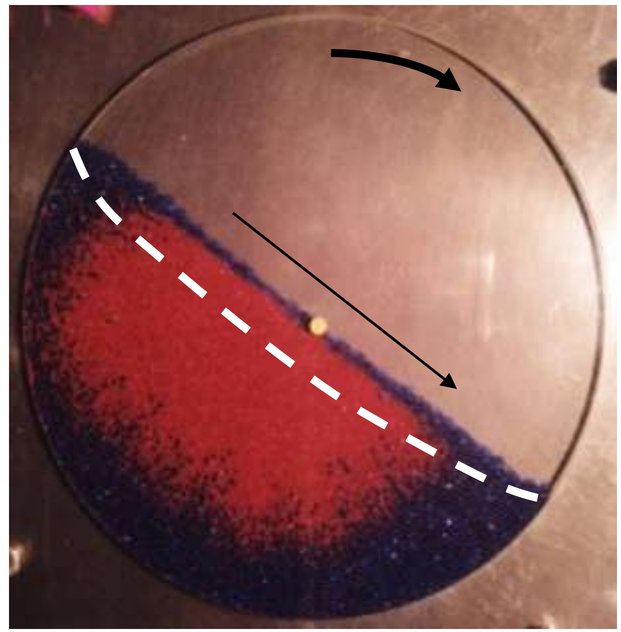
Low rotational speeds
Large size ratios



Streak formation

(Khakhar, Orpe and Ottino 2001)

High rotational speeds
Small size ratios



Core formation

(Hajra and Khakhar, 2004)

small
large

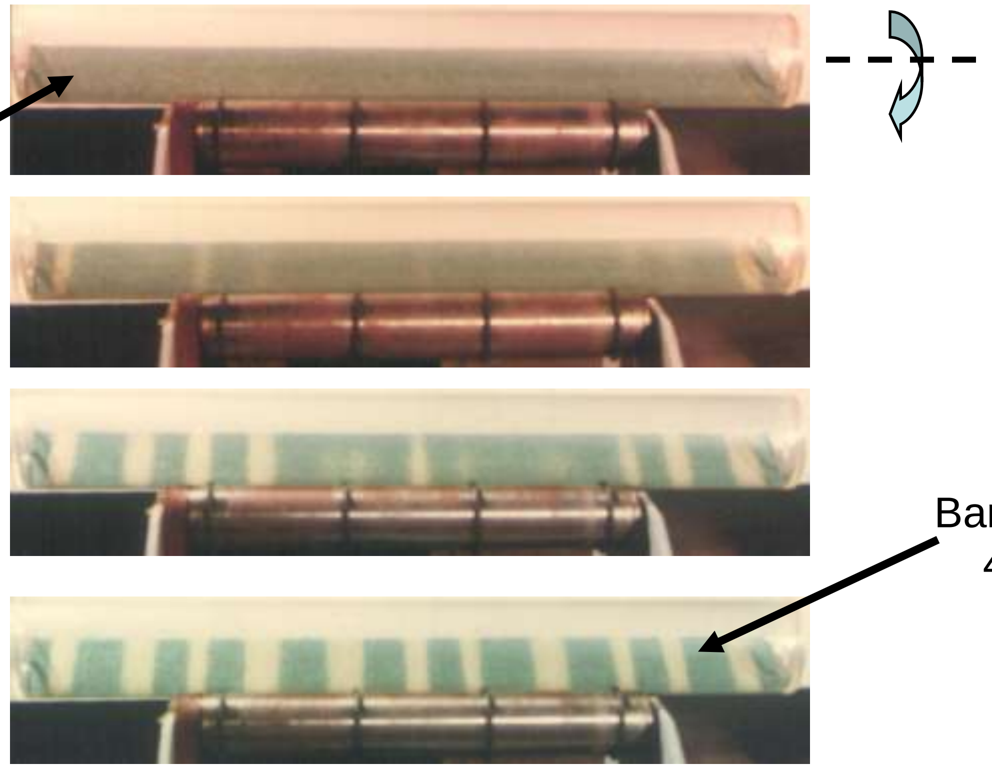
Segregation phenomenon occurs only in the thin flowing layer near the surface

Modeling flow in this thin flowing layer ?

Axial segregation in rotating cylinders

Spontaneous formation of bands in sand of two different grain sizes

Horizontal rotating cylinder



Initial uniform mixture

Band formation (30-40 revolutions)

(Das Gupta, Bhatia and Khakhar 1992)

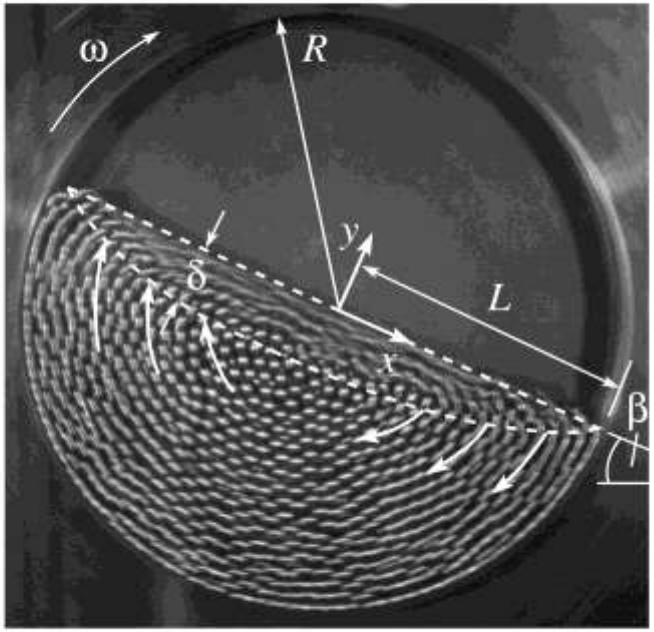
Flow of particles in axial direction within the thin flowing layer at the free surface due to dissimilar angles of repose for different sized particles.

Continuum model for surface flow

Assumptions (depth averaged equations)

Constant density (ρ) across the layer.

Linear velocity profile: $v_x = u\,(1 + y/\delta)$.



Stress equation (flowing layer)

$$\tau_{xy}\Big|_{y=-\delta} = -\rho c d \delta \left(\frac{dv_x}{dy}\right)^2 - \rho g \delta \cos\beta \tan\beta_s$$

Collisional stress *Frictional stress*

c ≈ 1.5 (fitted constant), β_s: Static angle of repose

$\tan\beta_s$: co-efficient of dynamic friction

Mass balance equation

$$u\delta = \frac{\omega L^2}{2}\left(1 - \frac{x^2}{L^2}\right)$$

Momentum balance equation

$$u\frac{du}{dx} = \frac{3g}{4}\frac{\sin(\beta-\beta_s)}{\cos\beta_s} - 3cd\frac{u^2}{\delta^2} + x\omega\frac{u}{\delta}$$

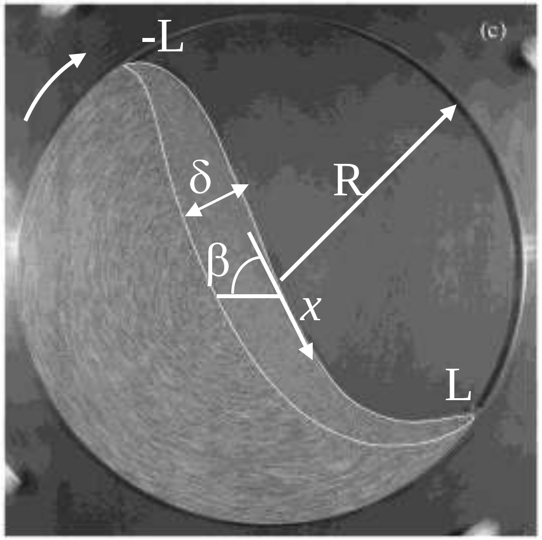
Net acceleration (*Gravity – friction*) *Collision* *In-flow/out-flow*

Khakhar, Orpe, Andresen, Ottino, J. Fluid Mech., 441, 255 (2001)

Flow scale measurements

Experimental System

Cylinders
Acrylic (R = 4, 8 and 16 cm)
Particles
Steel (spherical): 1, 2, and 4 mm
Glass (nearly spherical): 0.8, 1, 2 and 4 mm
Sand (aspherical): 0.4 and 0.8 mm
Operating conditions
Various rotational speeds



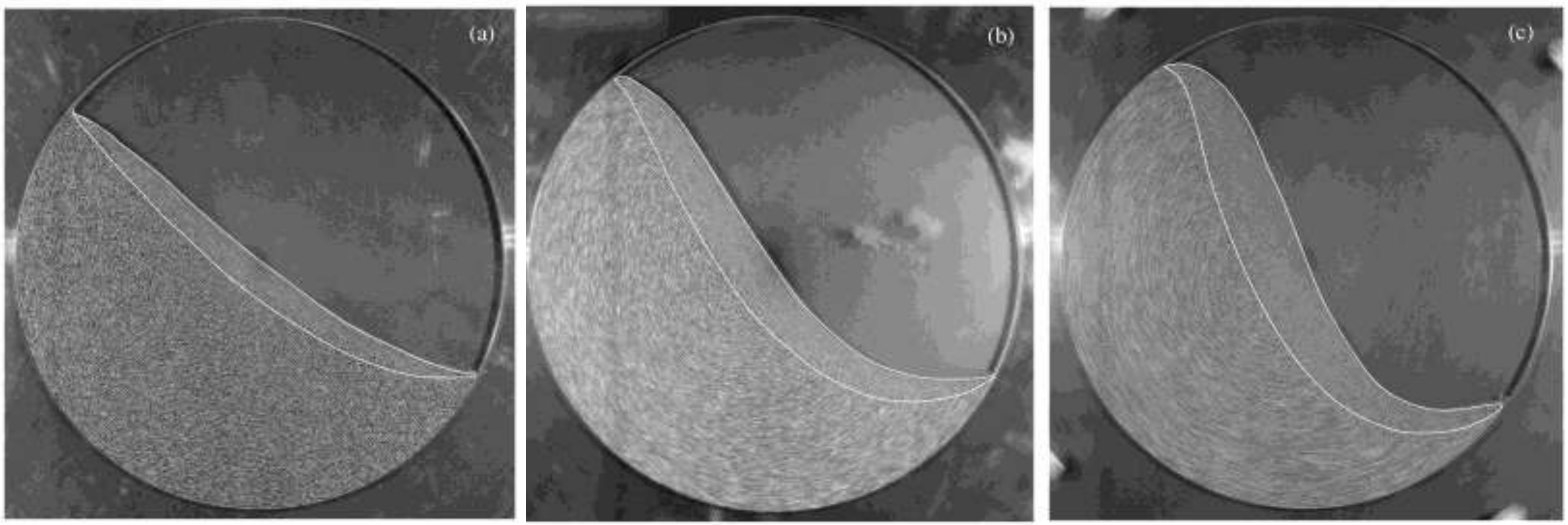
Experimental procedure :

- Digital images for different exposure times.
- Tracing of free surface and interface on images.
- Fitting polynomial (10^{th} degree) to traced curves.
- Computing $\delta(x)$ and $\beta(x)$ profiles from fitted polynomials.

Orpe and Khakhar, *Phys. Rev. E*, 64, 031302 (2001)

Effect of Rotational Speed

$Fr = 2 \times 10^{-3}$ $\quad$ $Fr = 22 \times 10^{-3}$ $\quad$ $Fr = 64 \times 10^{-3}$



2 mm steel balls in a cylinder with radius 16 cm

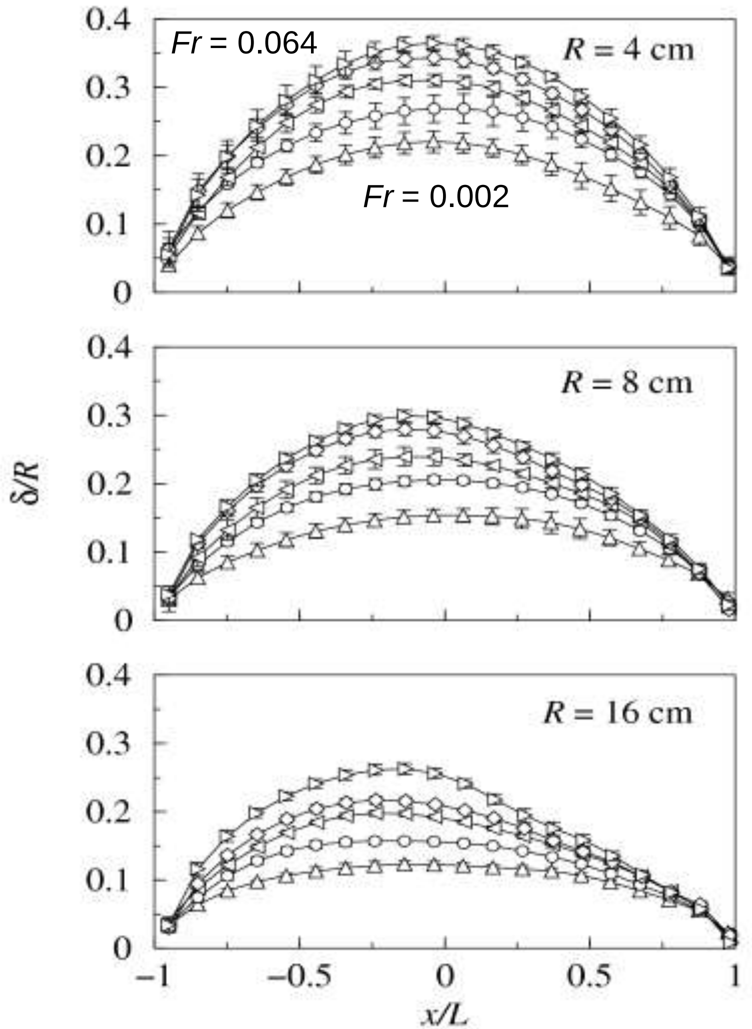
Froude number

$$Fr = \omega^2 R / g$$

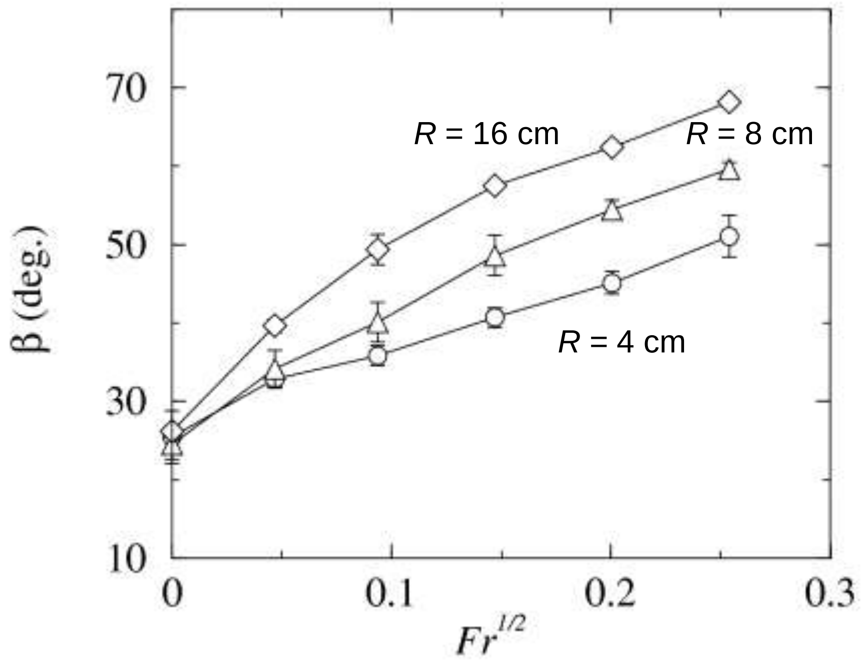
R : Cylinder radius
w : Rotational speed
g : Acceleration due to gravity

Measured Variables

Experimental data for 2mm steel balls



Layer thickness profiles

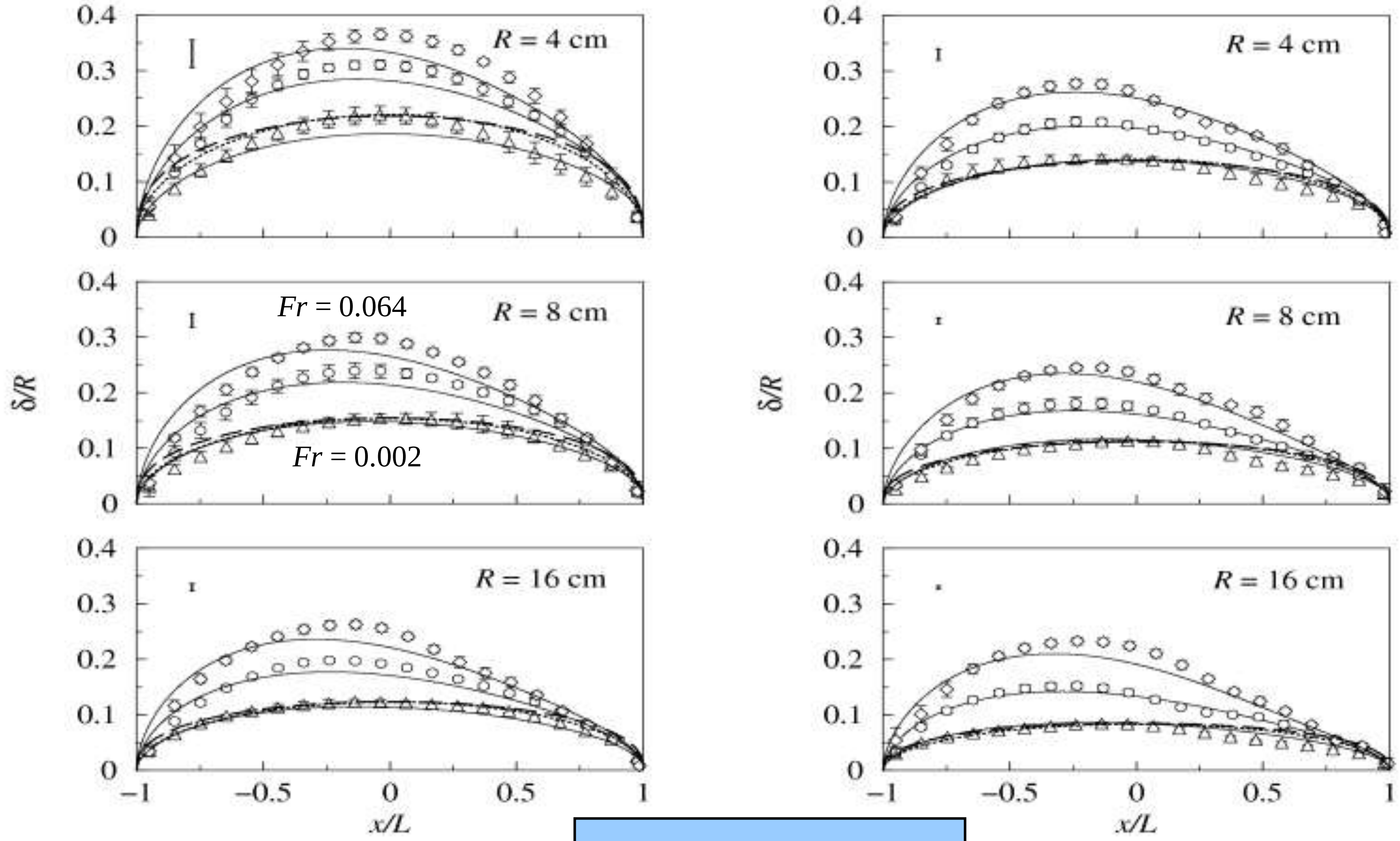


Surface angle at maximum layer thickness

Froude number (Fr) = $\omega^2 R/g$

Comparison to model predictions

Layer thickness profiles



2 mm Steel

0.8 mm Sand

Ranges of validity:

Fr ∈ (0.002, 0.064)

s ∈ (0.005, 0.05)

Steel, glass and sand

Model inputs: β, β_s and c

Continuum Model (Contd.)

Stress at the interface

(Mohr-Coulomb failure criterion)

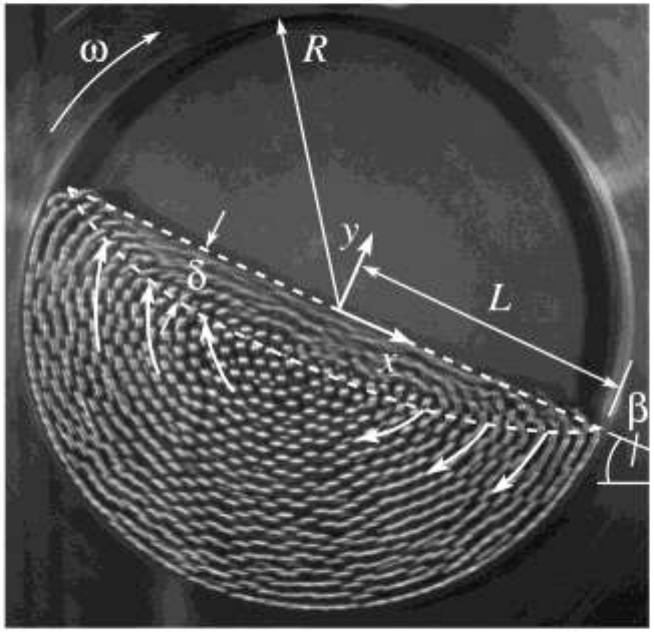
$$\tau_{xy}\Big|_{y=-\delta} = -\rho g(\delta + d)\cos\beta_m \tan\beta_m$$

β_m: Maximum angle of stability

$\tan\beta_m$: co-efficient of static friction

For d >> δ ⟹ $u = \dot{\gamma}\delta / 2$

(thick flowing layers)



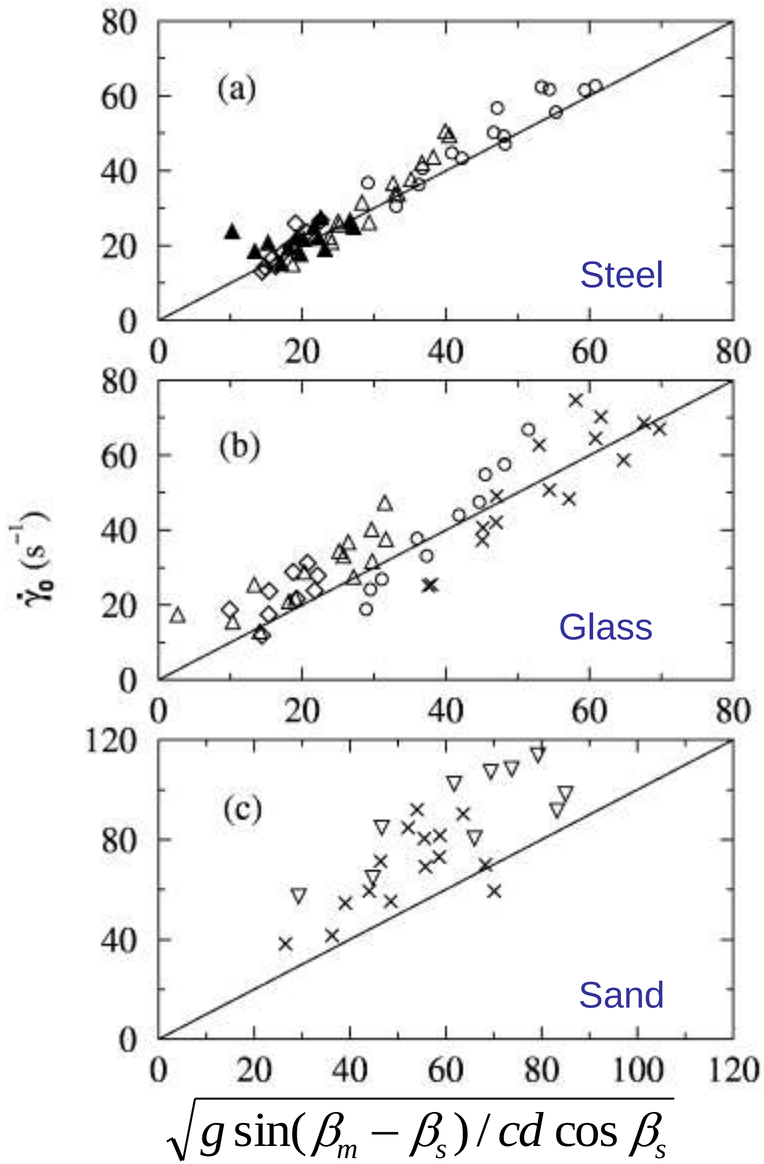
Characteristic Shear rate

$$\dot{\gamma} = \left[\frac{g\sin(\beta_m - \beta_s)}{cd\cos\beta_s}\right]^{1/2}$$

β_m, β_s: measurable quantities from experiments

Comparison to model predictions (Contd.)

Shear rate scaling

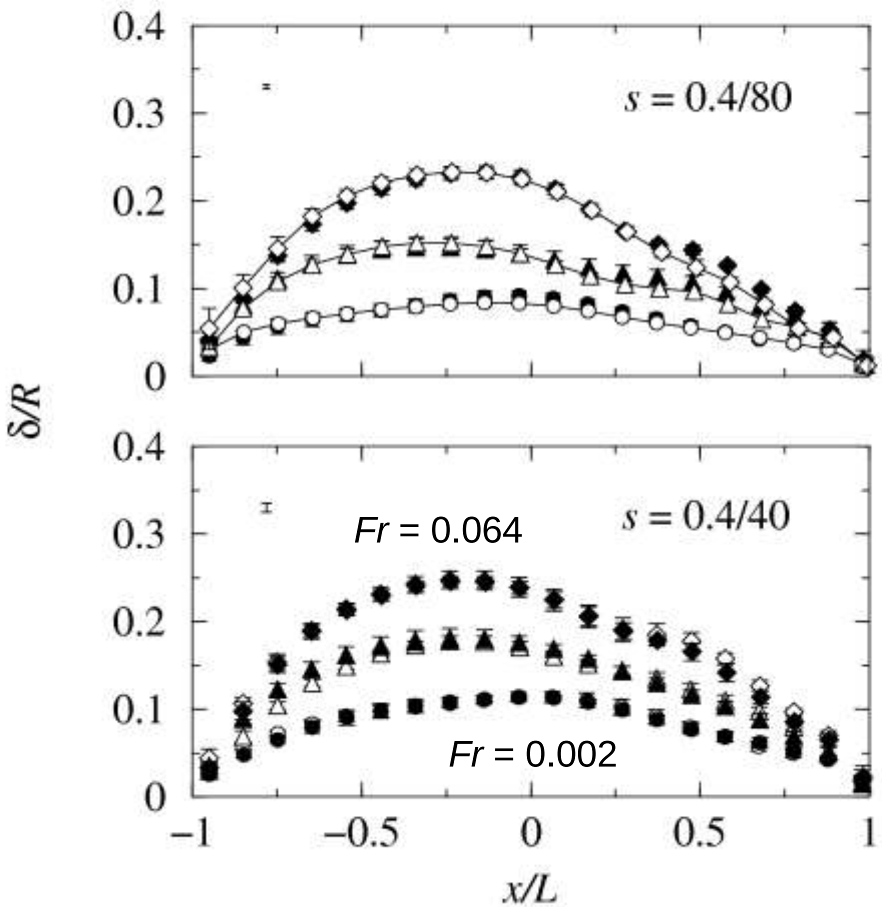


Model predicts shear rates in the flowing layer reasonably; ability to scale velocity profiles

Scaling for Sand

Layer thickness for equal Froude number (Fr) and size ratio (s)

$Fr = \omega^2 R/g$, $s = d/R$



d = 0.4, R = 80

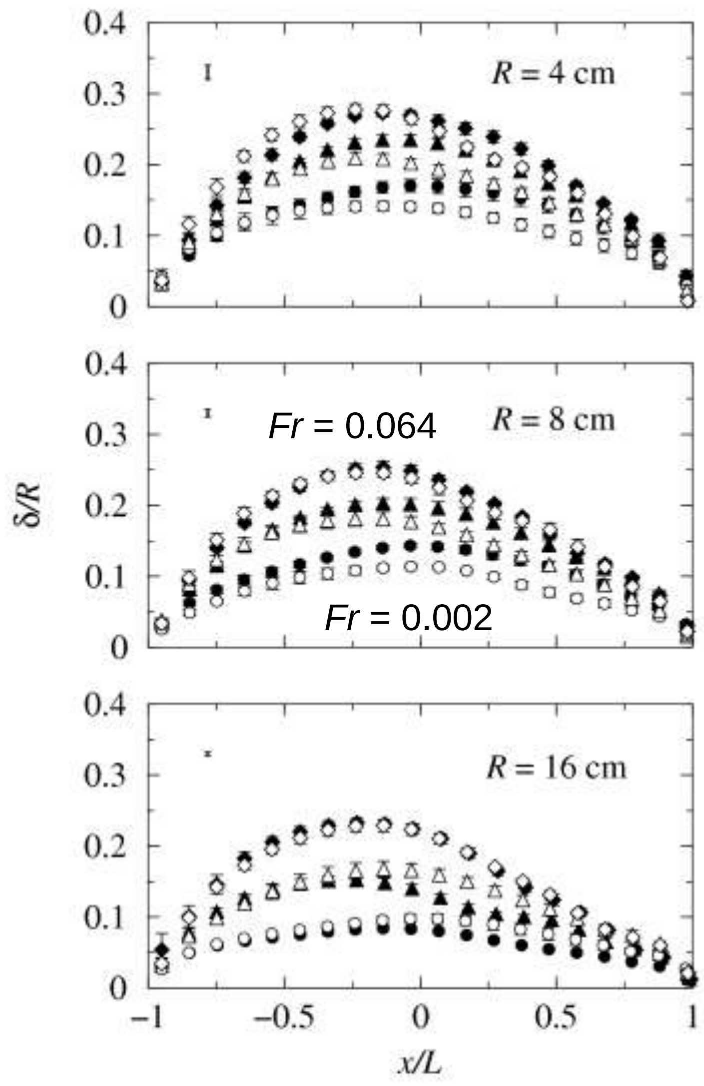
d = 0.8, R = 160

d = 0.4, R = 40

d = 0.8, R = 80

Scaling: Glass and Sand

Data for equal Froude number (Fr) and particle size (d = 0.8 mm)



Conclusions

- Surface flow in rotating cylinders depends primarily on Froude number (Fr) and size ratio (s).

- Flow behaviour nearly independent of particle materials studied.

- Continuum model predictions for layer thickness in good agreement with experimental results for a wide parameter ranges. Data fitting yields an empirical ($c \approx 1.5$) stress constitutive equation

$$\tau_{xy}\Big|_{y=-\delta} = -\rho c d\delta\left(\frac{dv_x}{dy}\right)^2 - \rho g\delta\cos\beta\tan\beta_s$$

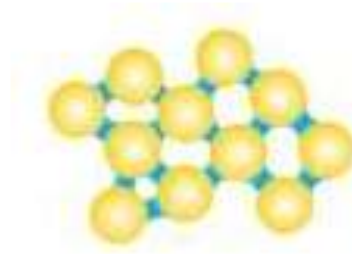
- Results useful for *scale-up.*

Flow of cohesive granular media

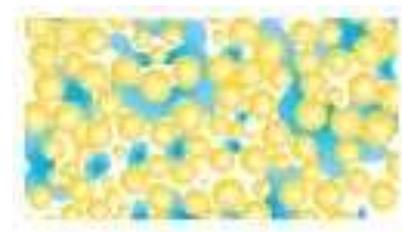
Partially saturated granular media

Features

- Presence of a small amount of interstitial liquid
- Formation of liquid bridges between particles
- Additional capillary and viscous forces



Liquid bridges between contact points



Large contiguous wet clusters

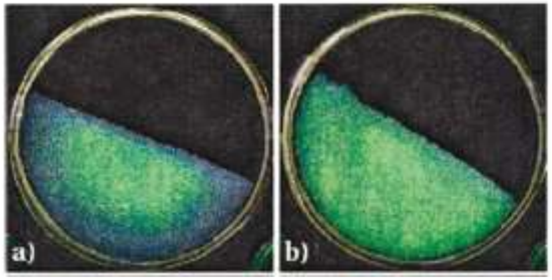
- Pile of wet material is more stable
- Failure in the bulk instead of at free surface
- Ratholing and jamming in hopper drainage
- Increased stability due to capillary bridges orientation

Hornbaker et al. 1997, Bocquet et al. 1998, Halsey and Levine, 1998, Samadani and Kudrolli, 2000, 2001, Nowak et al. 2005

- Tuning of mixing/segregation by addition of small amount of liquid

---- Li and McCarthy, 2003, 2005

Dry, segregated *Wet, well-mixed*

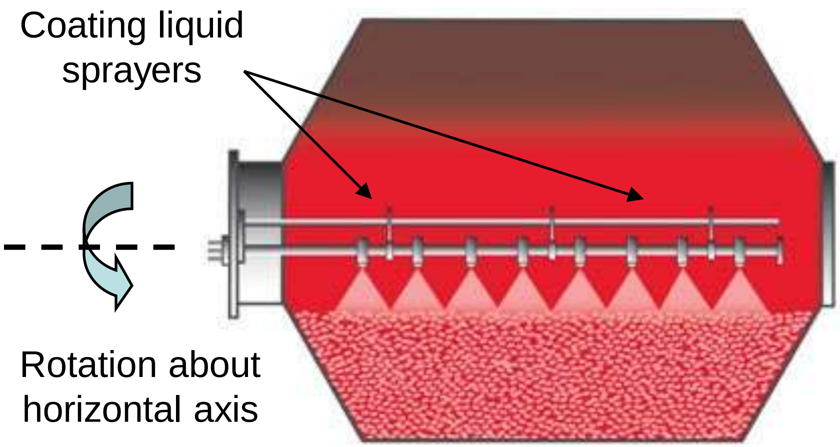


Cohesive powders in practice

Coating of tablets in pharmaceutical industries

To mask unpleasant taste or odor of the drug, protect it from environment, provide a means of identification, control bioavailability of the drug

Coating process



Horizontal drum filled with tablets/capsules

Repeated exposure to liquid spray

Final desired form of the tablets/capsules ready to be distributed

Understand the dynamics of particle flow for improving coating process

--- particle-liquid coupled flow, spreading of liquid over particles during flow, dynamics of drying of liquid over particle surface, exposure time of particles to spray zone

Experiments investigations

Cylinder:
Aluminium 29 cm diameter, 15 cm length

Particles:

1 mm spherical glass beads

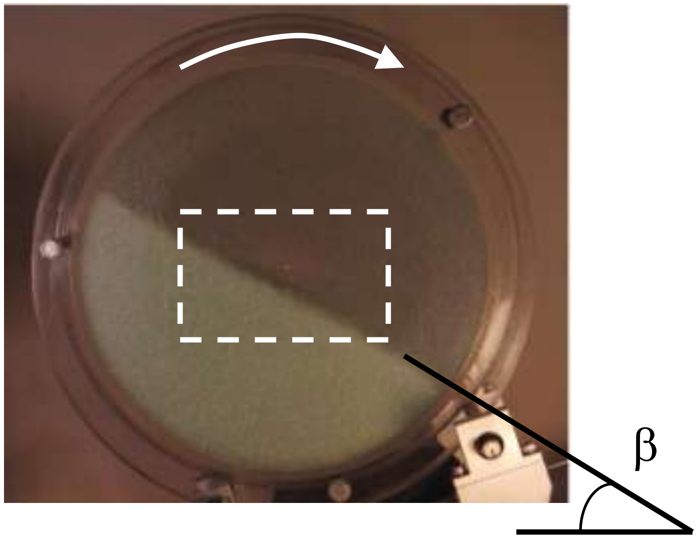
Liquid:

Silicone oil (ν: 0.05, 0.2, 1, 10 cm^2/s)

Operating conditions:

Rotational speeds: (0.00456 – 14 rpm)

Volume fractions of added liquid: (0.08 - 6) x 10^{-3}



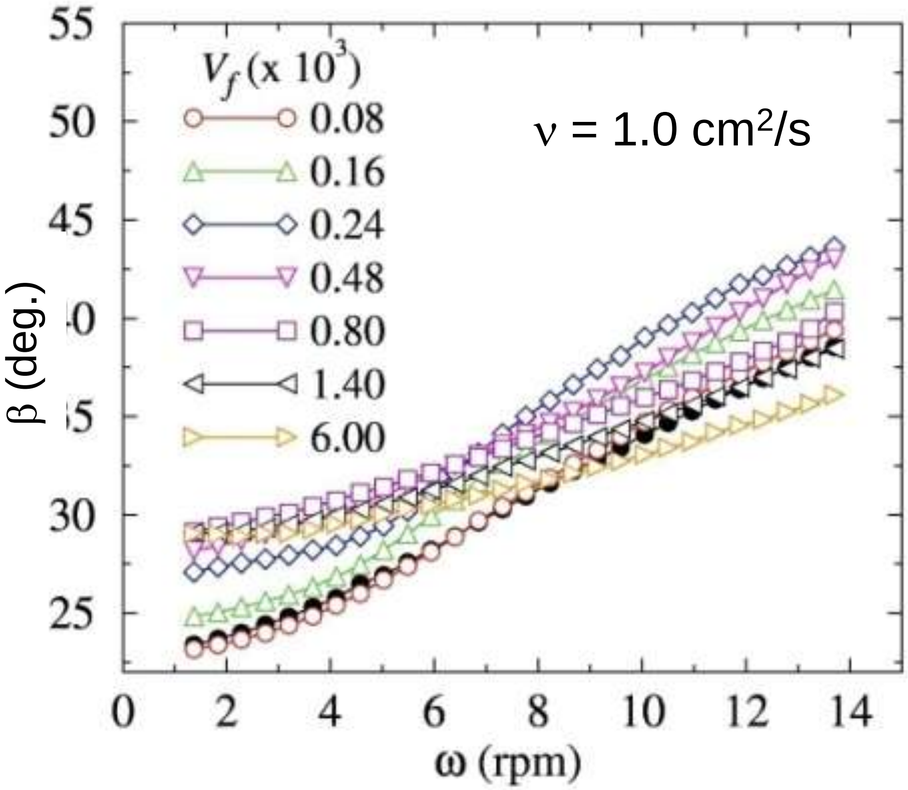
Experimental procedure :

- Acquiring digital images using back-lighting
- Tracing of free surface on digital images by edge detection
- Measuring angle of the detected edge of flowing layer
- Flow in continuous regime
- Measurements at the end wall

Xu, Orpe and Kudrolli, Phys. Rev. E, 76, 031302 (2007)

Dynamic angle of repose

Dependence on rotational speed



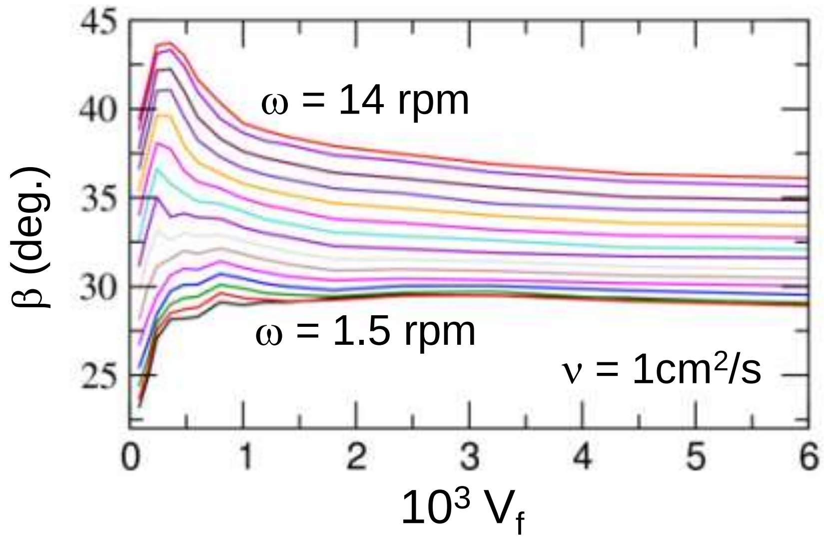
- Angles increase with rotational speed (ω)
- Rate of increase with rotational speed reduces for increased volume fraction
- Higher angles for wet system is most cases, with trend reversing at highest V_f and highest ω
- Non-monotonic changes in angle with increasing V_f

V_f = Vol. of liquid added / Vol. of dry beads

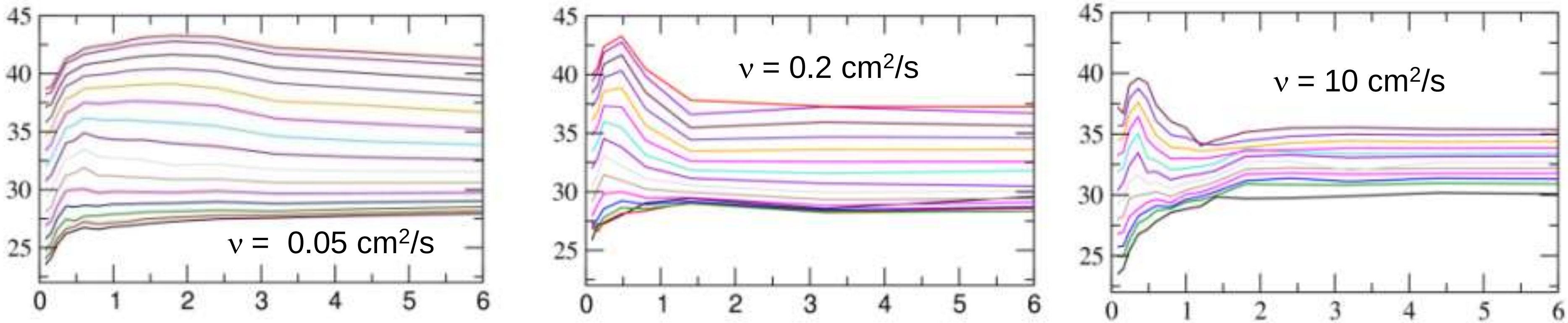
Filled circles: Angle of repose for dry system

Dynamic angle of repose

Dependence on volume of liquid added



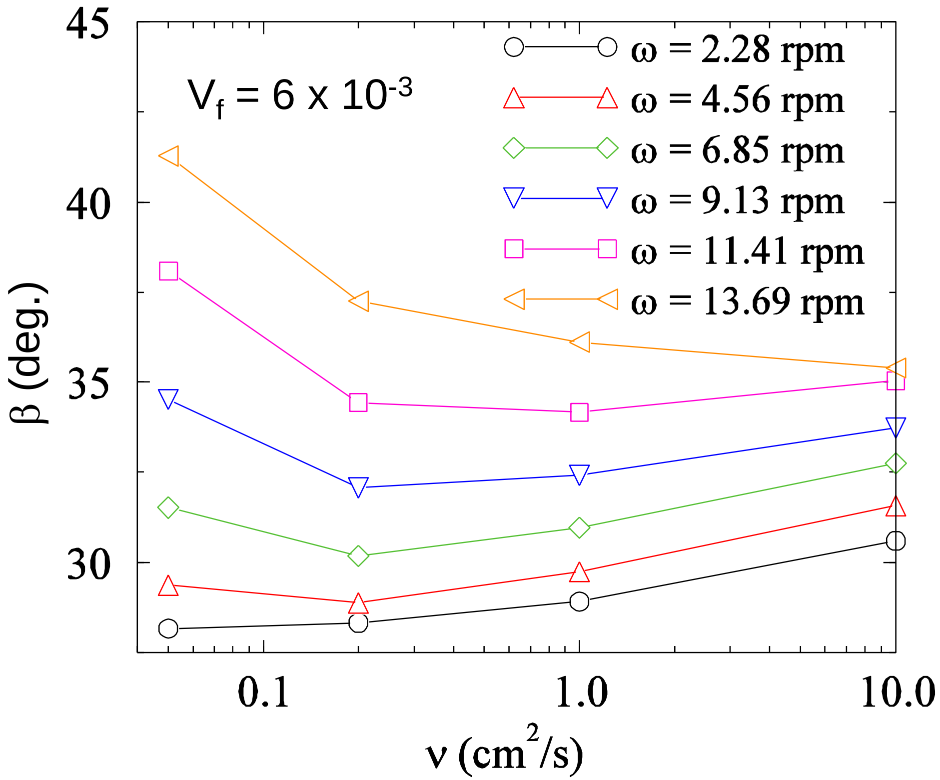
- Rapid increase with V_f for low values: increased cohesion between particles
- Steady decrease at high V_f: increased lubrication between particles
- Peak formation: competition between the two forces varying with V_f



Width of the peak formation narrows with increasing liquid viscosity: Varying interplay between cohesion and lubrication,

Dynamic angle of repose

Dependence on viscosity of added liquid

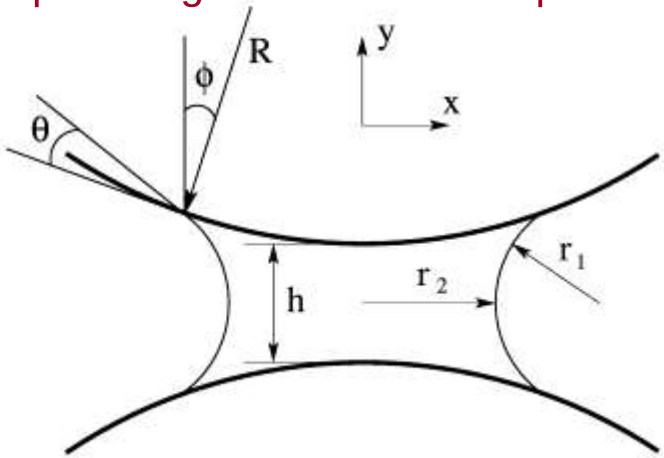


- Angles increase with viscosity at lower ω: increased resistance due to viscosity
- Decrease at higher ω: Viscous liquid assist the flow

Effect of added liquid is sensitive to rotational speeds employed: primary effect is to induce newer capillary attractive and viscous repulsive forces in addition to usual frictional and collision forces

Forces due to interstitial liquid

Liquid bridge connection two particles



Assumptions

- Negligible gravity effects (small amt. of liquid)
- Negligible curvature effects ($R >> r_2 >> r_1$)
- Cylindrical liquid bridge (flat profile)

Capillary (attractive) force

$$F_{cap} \approx \pi d \sigma \cos\theta \left[1 - \frac{1}{\sqrt{1 + \frac{4V}{\pi d h^2}}} \right]$$

$V = \pi d[H^2(b) - h^2]/4$

$H(r) = h + 2r^2/2$

b: wetted area, σ: surface tension, V: volume of liquid bridge

Viscous (repulsive) force

$$F_{vis} = \frac{3}{8} \pi \rho_l \nu d^2 \left[1 - \frac{h}{H(b)} \right] \frac{1}{h} \frac{dh}{dt}$$

ρ_l: density of liquid, dh/dt: rate of particle approach towards each other

Pitois et al., J. Coll. Int. Sci., 2000

Flow induced particle motion time scales

Approach: Calculate appropriate time scales corresponding to motion of particles with respect to each other, based on velocity/layer thickness estimates

Mass Balance in flowing layer → $\bar{u}\bar{\delta} = \frac{1}{2}(1-\xi^2)$ → $u = \frac{\omega R_0{}^2}{2\delta}$ { x = 0; center; R = L; low ω }

$$\xi = x/L, \bar{u} = u/\omega L, \bar{\delta} = \delta/L$$

Time require for two particles to travel past each other

$$t_x = \frac{d}{u} = \frac{2d\delta}{\omega R_0{}^2}$$

{ δ is weakly dependent on ω and V_f → t_x primarily depends on ω }

Time require for two particles to come close to each other by squeezing out the liquid

$$t_y = \frac{9}{4}\frac{\nu\rho_l}{dg\rho_b}\left[\frac{2b^2 d(h_s - h_0)}{(2b^2 + h_s d)(2b^2 + h_0 d)} + \ln\left(\frac{1 + 2b^2/h_s d}{1 + 2b^2/h_0 d}\right)\right]$$

(balancing viscous force to weight of particle and integrating); h_s: surface roughness of beads

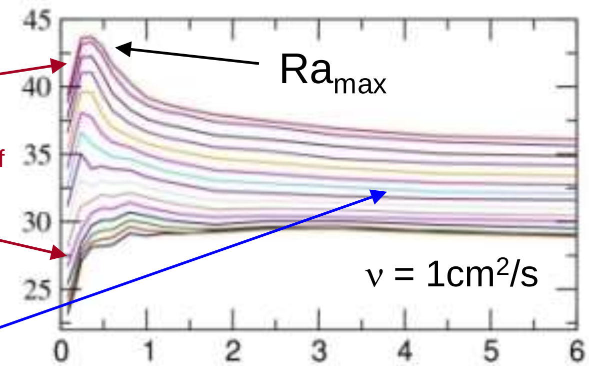
Relative significance of liquid induced forces

Ratio of the time scales: $$Ra = \frac{t_x}{t_y} = \frac{0.13\delta/(\omega\nu)}{\left[\frac{2b^2 d(h_s - b)}{(2b^2 + h_s d)(2b^2 + bd)} + \ln\left(\frac{1 + 2b^2/h_s d}{1 + 2b/d}\right)\right]}$$

For small V_f, b ~ h_s → Ra >> 1

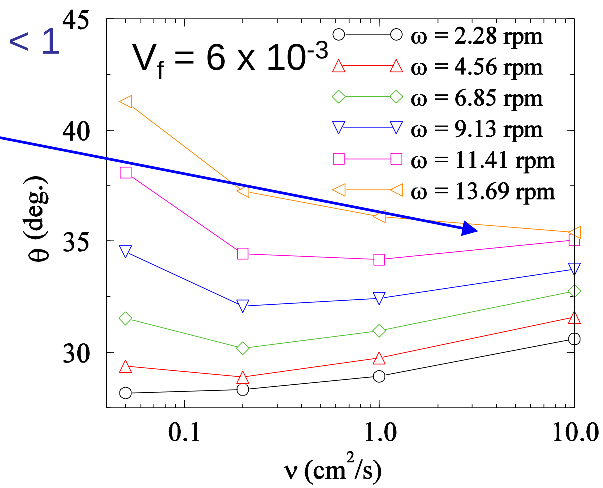
Uniform coating of liquid over particle surface

Initial rise with V_f



For large V_f, b ~ d/2 → Ra → $10^{-2}/\omega\nu$

Ra ~ O(10^{-1}) < 1



Analysis explains many of the observable experimental features

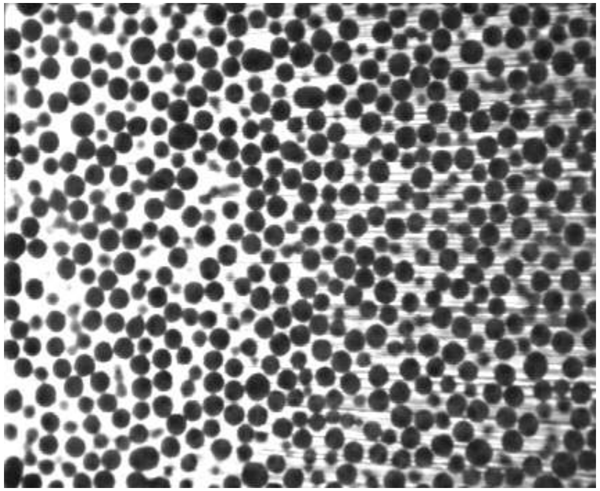
Certain unexplainable effects need more rigorous study involving accurate estimates of competing forces

Conclusions

- Systematic investigation of the effect of rotational speed, interstitial liquid volume fraction and viscosity on the values of dynamics angle of repose
- Non-linear increase in the angle of repose with rotational speed. Angle of repose is higher in presence of interstitial liquid than for dry system.
- Initial rapid increase in the angle with increasing volume fraction of added liquid followed by a decrease at higher volume fraction.
- Simple time-scale analysis of relative motion of two adjacent particles in flow serves to explain most of observed non-intuitive behaviour.
- Need for a more rigorous study to
  --- account for unexplainable observations
  --- account for accurate estimates of several competing forces
  --- extend continuum model to account for liquid induced forces
  --- investigate applicability of analysis for non-spherical particles (e.g. coating process of pharmaceutical tablets)

Interior imaging of dense granular media

Particle flow study in the bulk



Non-invasive techniques to measure flow in the bulk

- Refractive index (RI) matching
- X-ray tomography

} Identifying and tracking of every particle within region of interest

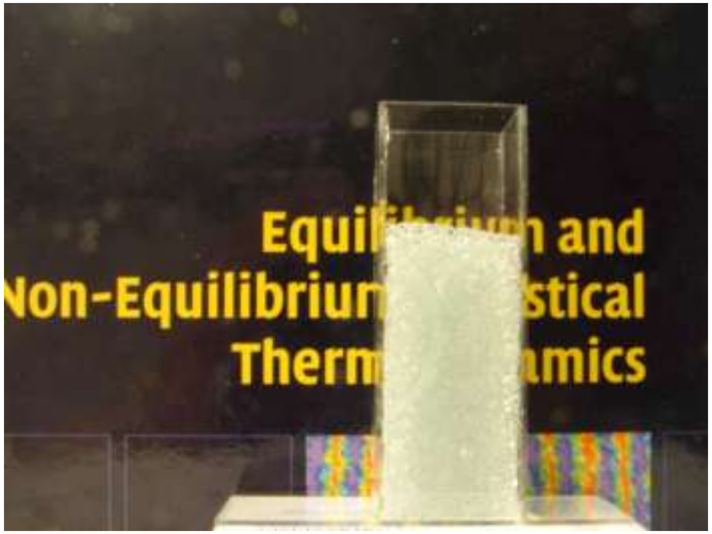
- Magnetic resonance imaging (MRI)
- Positron Emission Tracking (PET)

} Use of tracer particles: average flow measurement at a given position

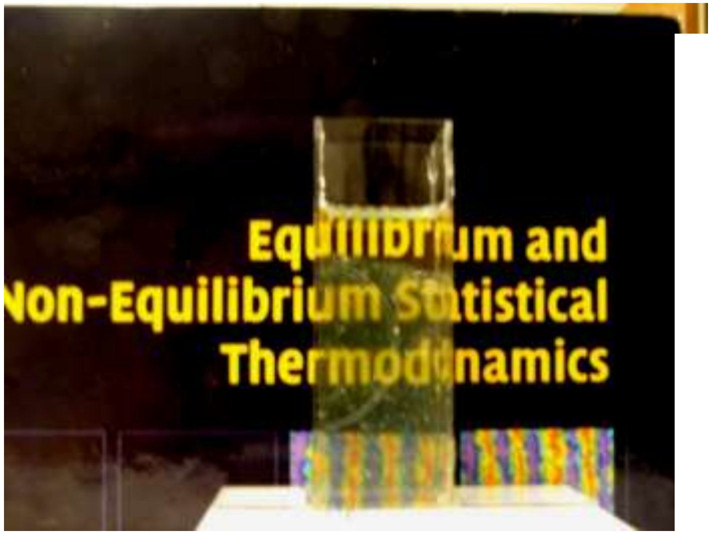
RI matching technique more easy to set-up, capable of local flow measurement at very high accuracy, ideal to carry out micro-rheology studies in a given system → Effect of viscous interstitial liquid ?

Particle imaging technique

Refractive Index matching



Bin filled with beads



Bin filled with beads and a liquid with the same refractive index

Particles : glass beads (d = 1mm)

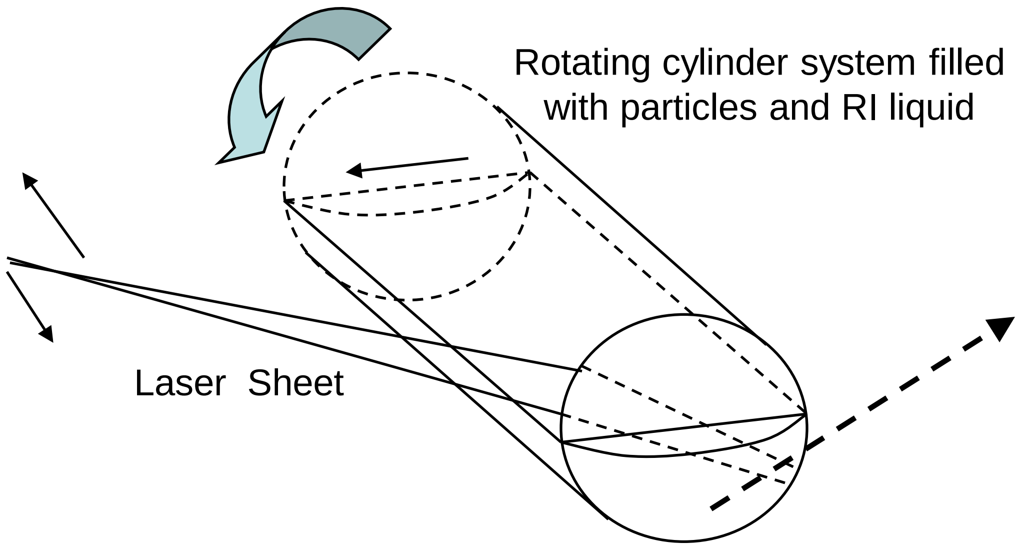
Liquid (HC oil) : RI ~ 1.52, μ ~ 25 cP

Gollub et al (2003), Pouliquen et al. (2003), Siavoshi et al (2006), Losert et al (2006), Orpe et al. (2007, 2008)

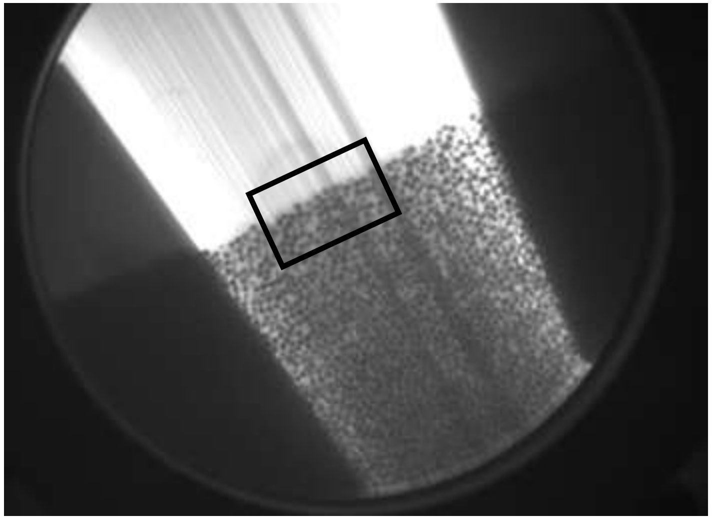
Rotating cylinder flow

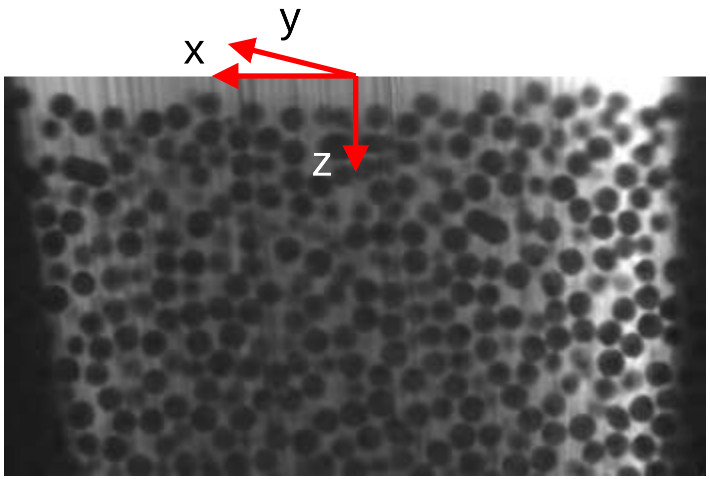
Experimental: set-up, imaging, procedure

Laser induced fluorescence (LIF)



10 particle diameters inside





Cylinder: Acrylic 8 cm diameter, 15 cm length

Particles: Glass beads (spherical): 1 mm

Refractive index Liquid: μ = 25 cP

Operating conditions: ω : (2 - 10 rpm)

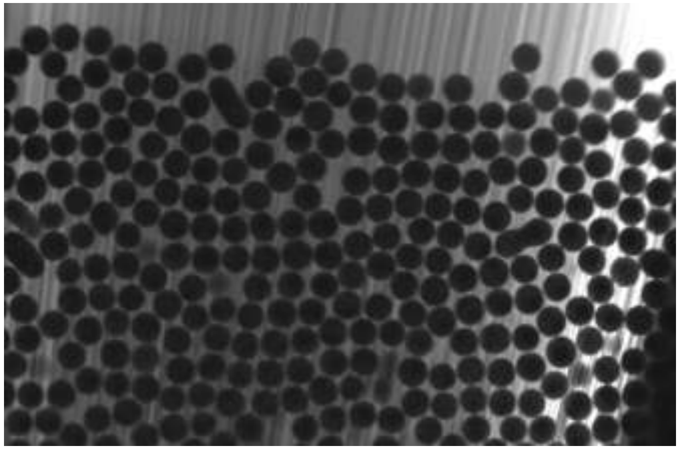
Imaging: Different depths (y) from one end wall

Particle identification/tracking using IDL (*Crocker 1996, Tsai et. al, 2003*)

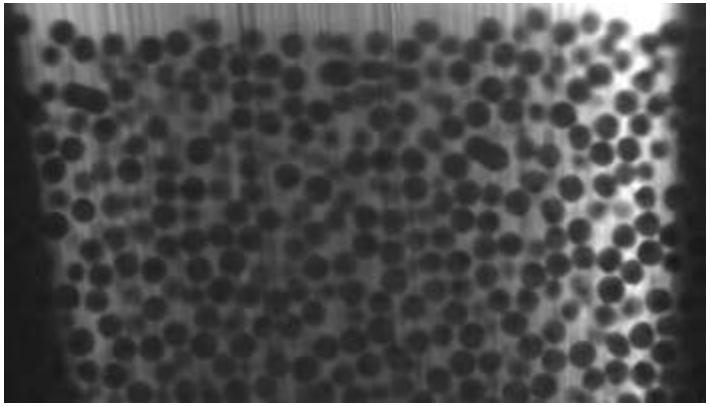
Mean velocity profiles

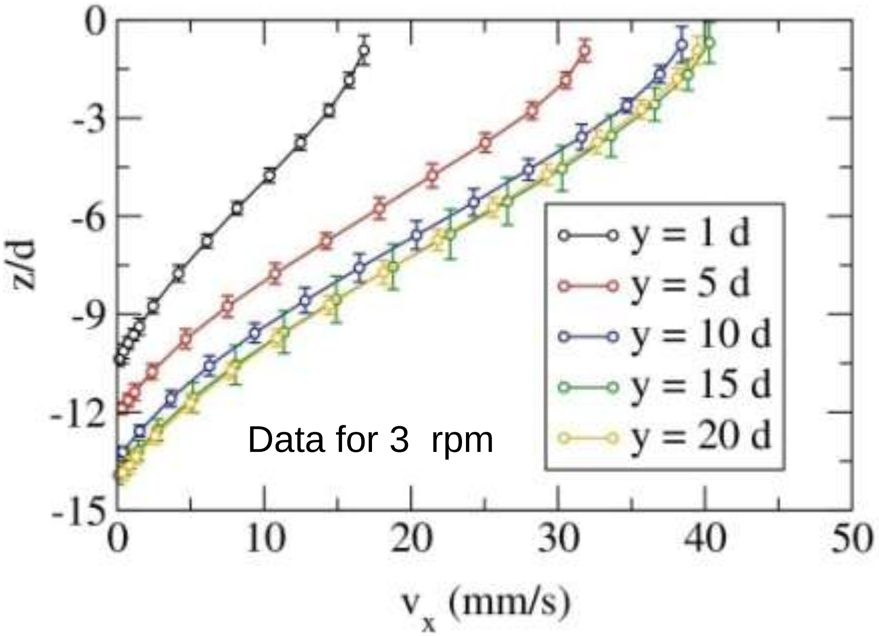
Effect of end-walls

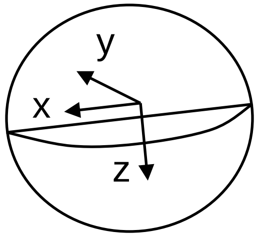
Flow near the wall



Flow 10 particle diameters inside



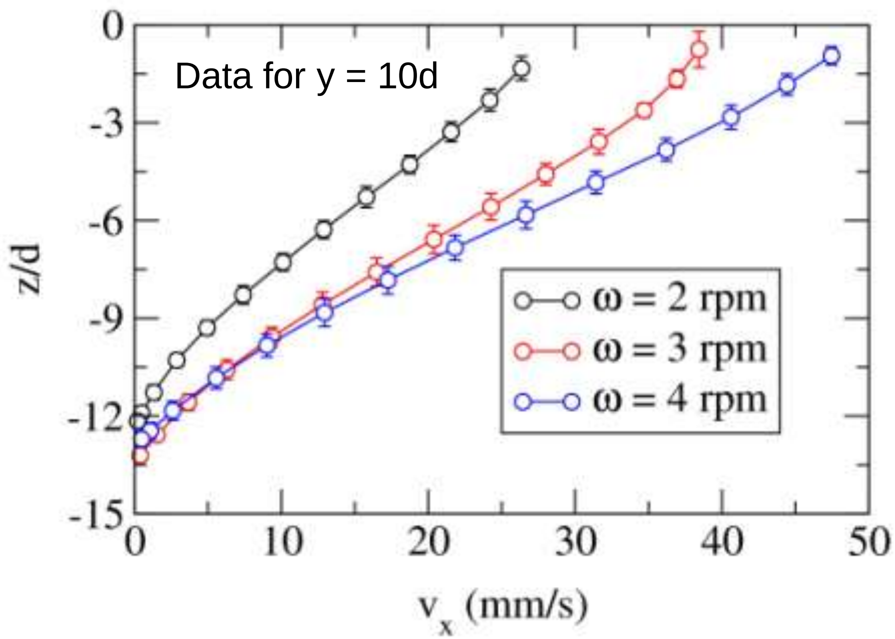


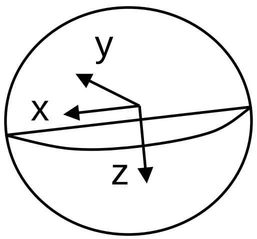


Wall effects penetrate upto 10 particle diameters; shape of velocity profiles same everywhere

Mean velocity profiles

Effect of rotational speed



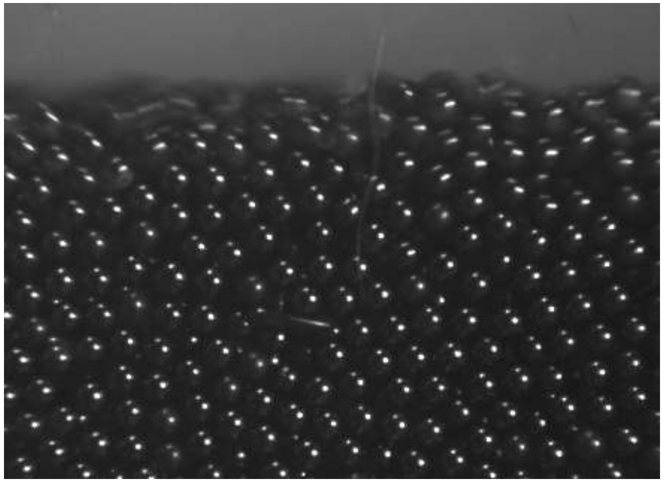


Velocity magnitude increases with increase in the rotational speed keeping the profile shape same

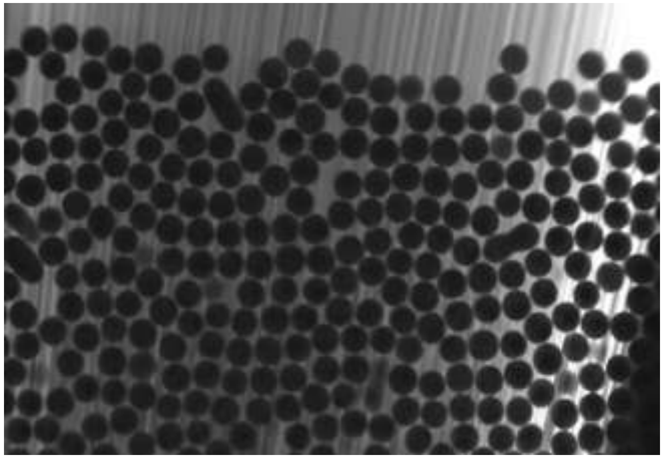
Mean velocity profile

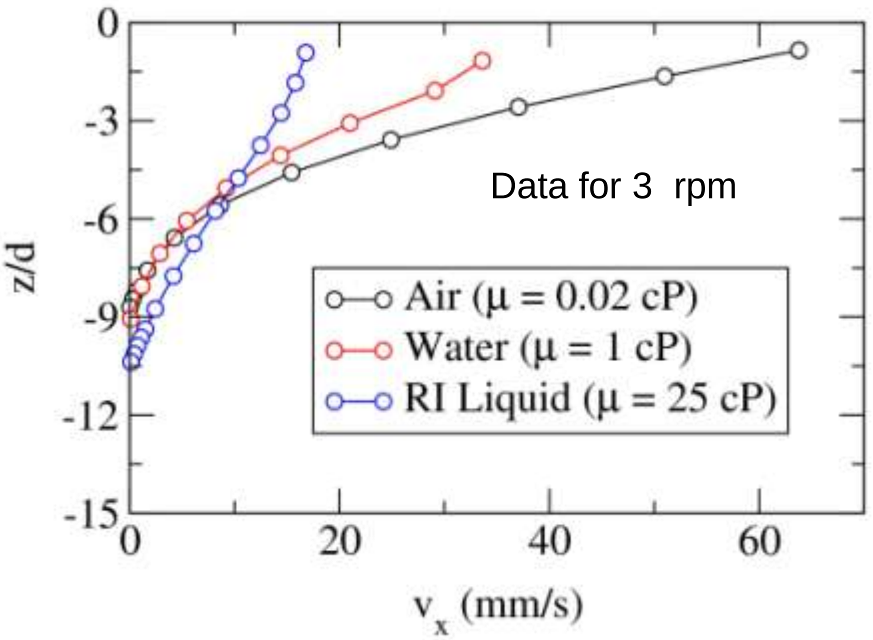
Effect of interstitial liquid

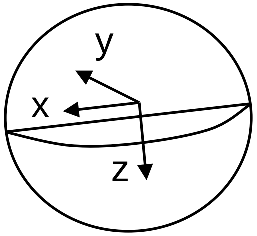
Flow for dry system



Flow in presence of liquid







Velocity magnitude decreases with increase in the interstitial fluid viscosity

Velocity profile scaling

Characteristic Shear rate:

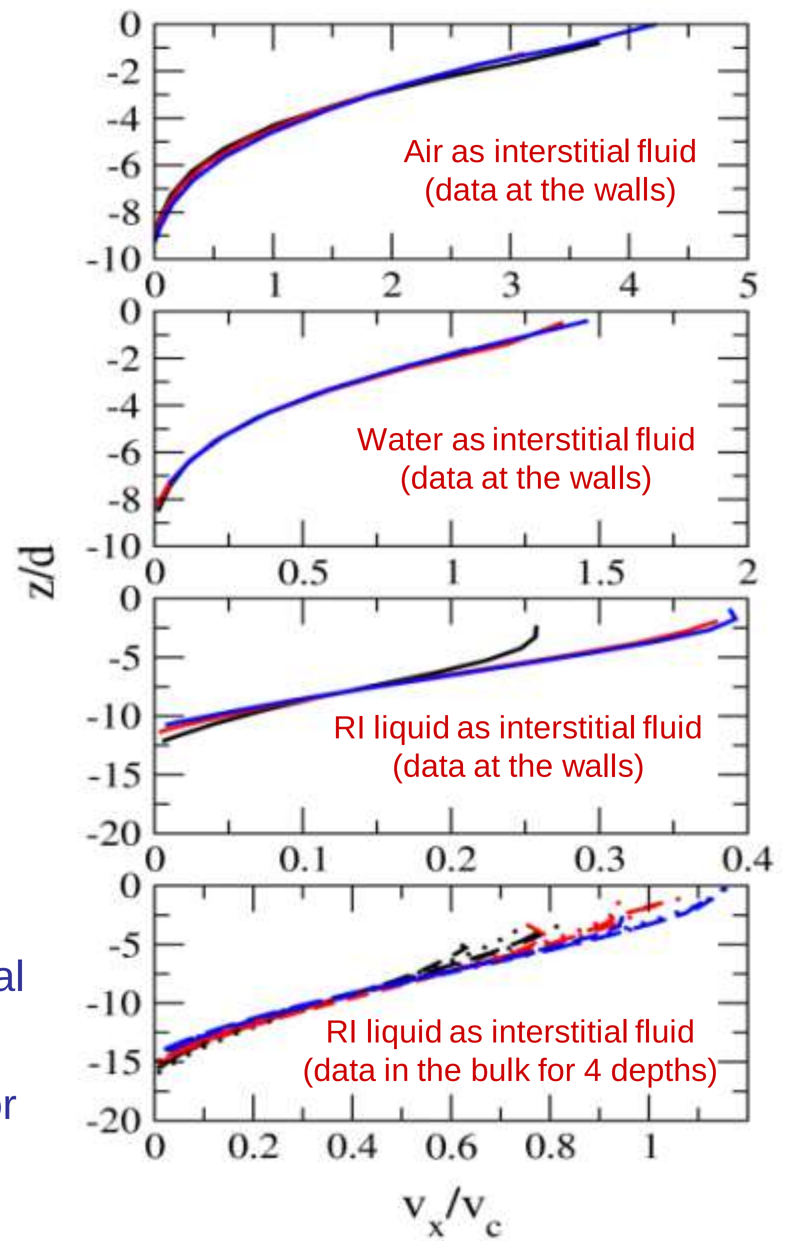
$$\dot{\gamma}_c = [g \sin(\beta - \beta_s) / d \cos \beta_s]^{1/2}$$

net driving force/mass: $g\sin\beta - g\cos\beta\tan\beta_s$.
(gravity – friction)

Characteristic length: *d*

Characteristic velocity:

$$v_c = [gd \sin(\beta - \beta_s) / \cos \beta_s]^{1/2}$$



- Model able to capture the essence of the flow mechanism in the presence of a viscous interstitial liquid
- Modifications needed in the model to account for the varying time-scales of particle motion due to varying viscosity of interstitial liquid

Conclusions

- Investigation of sheared dense granular flow system using internal imaging.

- Simple continuum model derived for dry systems seems to carry over to wet system atleast qualitatively

- Need for a more rigorous study to
  --- obtain flow details in axial direction
  --- account for varying time-scales of particle motion for a given liquid viscosity
  --- extend study to mixtures of particles, particles with non-spherical shapes
  --- include appropriate additional terms in the continuum model to achieve quantitative comparison at higher shear rates

Concluding remarks

What do we have at present ?

- **Continuum model**

  --- phenomenological, capable of predicting several facets of flow behaviour for spherical particles of various sizes in different system sizes and in presence of different interstitial liquid

  --- can be extended to explain flow behaviour of wet granular media by incorporating appropriate liquid induced forces

  --- simple scaling rule based on particle to system size and Froude number.

  --- can predict evolution of atleast one type of segregation behaviour (streak formation: Khakhar, Orpe & Ottino, Pow. Technol. 2001)

- **Visualization technique**

  --- carry out detailed micro-rheology by visualizing motion of each particle at the walls as well as the interior in various geometries

  --- can investigate flow of the viscous interstitial liquid

  --- can visualize formation/breakage of capillary bridges

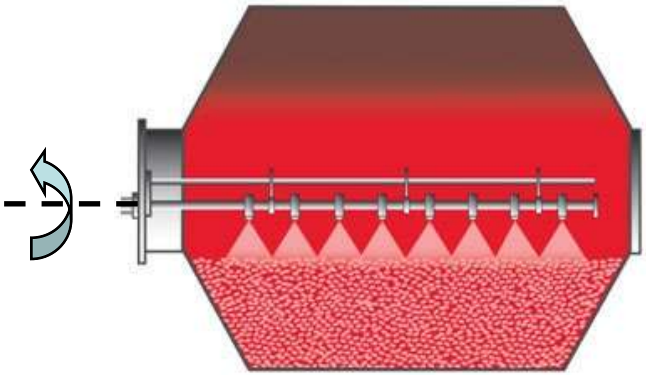
  --- definitely extendable to particles of irregular shapes (elliptical, triangular etc.)

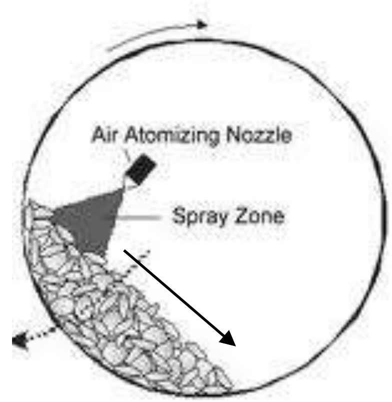
  --- easily usable for different complex systems, viz. slurry and suspension flows

Concluding remarks

What can we hope to achieve ?

Coating of pharmaceutical tablets





Issues (pertaining only to tablet handling)

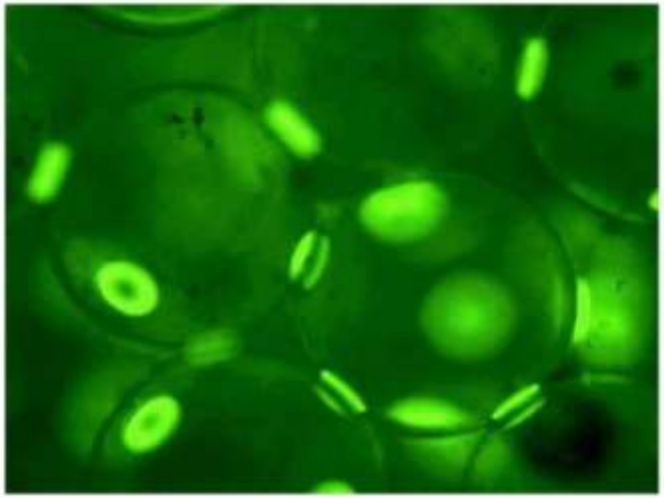
- Uniform exposure of every tablet to the spray zone, i.e. uniform re-circulation of each particle while in flow
- Dependency of the circulation time/exposure on system size, fill level, cylinder speed, bulk density, liquid viscosity and surface tension i.e. need for appropriate scale-up

Possible approaches

- Use of our continuum model to predict (a) flow rate of particles along the layer (b) average time of circulation in the cylinder (c) flow behaviour for changing cylinder size and rotational speed (d) probability of any particle to be within a spray zone as a function of cylinder size. (e) dependency of the flow behaviour on the fill level
- Derive new scaling relations taking into account fill level, constant particle size, varying cylinder size, varying rotational speed.
- Use imaging techniques (using tracers) to validate the predictions initially for spherical particles and later on for actual tablet shapes

General Conclusions

- Many fundamental discoveries are potentially possible
- Application to engineering systems could produce significant improvements in products and processes



Visualizing liquid bridges between particles inside the system --- Fournier et al. 2005

Granular matter is an exciting research area

Thank You